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ENGINEERING DESIGN WITH DECISION SUPPORT: AN APPLICATION OF GOAL DECOMPOSITION

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The engineering design problem of large systems is characterized by a hierarchical decomposition because of the complexity of the design process. This paper examines a modeling technique called Goal Decomposition that accommodates a hierarchical decomposition, and is therefore useful in the design of large systems. Two applications of goal decomposition are included to investigate the strengths and weaknesses of this technique.

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ENGINEERING DESIGN WITH DECISION SUPPORT: AN APPLICATION OF GOAL DECOMPOSITION

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SUMMARY

Mathematical modeling is becoming a vital tool in engineering design. These models have been used to provide decision support to designers of systems characterized by many design choices. However, for the design of complex systems, these models become too large and cumbersome to be of significant value. As a result, research has progressed in the decomposition of the design problem.

A decision support system, serving as a computer-based aid for engineers designing weapon systems, must integrate "procedures dealing with designing for producibility and designing for supportability with those dealing with designing for performance cost and schedule" (ULCE Implementation Plan, 1987). One of the characteristics of large weapon system development that causes difficulties in achieving this integration is the hierarchical nature of the design process. Typically, a system is designed in terms of subsystems which are required to meet certain specifications. Problems begin to arise when these specifications cannot be met. Consequently, these design processes may be enhanced by a computer-based system that supports the hierarchical nature of design.

This research examines the application of mathematical modeling and knowledge-based system techniques to the development of decision support systems to support engineering design.

PREFACE

The purpose of this technical paper is to document work conducted as part of the 1987 United States Air Force (USAF) - Universal Energy Systems (UES) Summer Faculty Research Program (SFRP) and subsequent mini-grant. This research was conducted in support of Air Force Human Resource Laboratory (AFHRL) involvement in the development of a prototype Unified Life Cycle Engineering (ULCE) system. The intent of this system is to serve as a computer-based aid for engineers designing weapon systems, in particular integrating procedures to address producibility and supportability in weapon system design.

I wish to thank AFHRL for their sponsorship and UES for their attention during the SFRP. Several individuals deserve special identification for their support of my work at AFHRL. Lt. Col. Joe Coleman was instrumental in helping me to identify an excellent research direction. Capt. Tom King provided timely resources and support. Maj. Joe Nerad, Capt. Maureen Harrington and Lt. Lee Dayton helped me several times in overcoming hardware/software difficulties.

I greatly appreciate the hospitality that I received from Bert Cream and Col. Donald Tetmeyer. I would also like to thank Richard Lamb, Jim McManus, Janet Peasant, Capt. Clem Schram, Connie Thompson, and Capt. Bill Weaver for making an enjoyable working environment and for making my stay a learning experience.

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1. INTRODUCTION

Engineering design in its broadest sense refers to the activity of selecting and placing materials to form a system. Because of the inherent complexities, design engineers have increasingly depended upon mathematical modeling to help guide the design process [12]. However, for the design of large systems, these models quickly become too large and cumbersome to be of significant value. As a result, research has progressed into the decomposition of the design problem with the intent of breaking the main design into a series of smaller design problems that can be solved and then recombined in the hope of generating a good global design.

Figure 1 illustrates a network model of the design decomposition problem. The node at the top level (level 0) of the network represents the system which is to be designed. As we pass downward through the network, the parts of the system are exploded into greater detail, until we reach the bottom of the network where no further decomposition is possible.

The network shown in Figure 1 can be thought of as actually containing two different types of optimization problems: (1) A high level optimization, and (2) Several lower level optimizations. The high level optimization is equivalent to answering the question, "What type of system do we want and what

do we want it to be able to do?" An example of this type of optimization problem, which is typically characterized by multiple objectives, can be found in [5]. In the case that several designs have already been established, Pareto optimal designs can be determined by the method given in [11] to reduce to list of candidate designs to some managable number. The design decisions that are made at this level are then passed downward to the next level (level 1) in the form of system specifications.

At the middle and lowest levels in the network, the objective is not to optimize the design of the subcomponents over some set of objectives, but instead to meet some stated level of attributes that has been provideu 'y a parent optimization. In some cases it may be suboptimal for a subcomponent to over-perform as well as under-perform. For example, in designing an aircraft, we would like to achieve a specified level of speed (an attribute measuring performance); however, optimizing the wing design for speed would clearly be suboptimal if the landing gear of the aircraft is not designed to accommodate the greater speeds.

Of course, other objectives besides performance may be of interest. Reliability, cost, supportability and producibility are some objectives that frequently must be addressed in designing systems. In each case, a stated level of achievement is what is required at the subsystem level.

The need for communication between levels in the hierarchy of Figure 1 is currently satisfied by multiple design iterations in real world design processes. In fact, it must be emphasized that a design of a large, complex system is characterized by many

design changes through iterations. As a result, decision support for design decomposition should accommodate these design alterations brought about by new information passed between levels of the network.

1.1 The Rationale Behind Decision Support

Decision support for the design process has several appealing functions besides potentially improving the ultimate system design. A decision support system (DSS s' uld provide greater managerial control of the design proces   example, a DSS should be a mechanism for coordinating sut /s em designs. This would allow the designer to recognize when a particular subsystem design is insufficient to meet system needs, and what the best alternative designs are. This feature would generate critical information early in the planning phase, and therefore direct management's attention to the important design issues. In addition, using a support system would allow the designer to determine the system degradation due to not meeting a specific system specification.

The purpose of this paper is to describe a goal decomposition approach that can be useful in a DSS for the planning phase of engineering design. In the following section we describe some of the approaches that currently exist for decision support of the design process. In section 3, a goal decomposition method is examined and related to the network in Figure 1. In addition, some refinements to the goal decomposition algorithm are suggested. Section 4 contains two

example applications. Section 5 contains the concluding remarks.

2. DECISION SUPPORT FOR THE DESIGN PROCESS

The engineering design problem is typically expressed as:

(P1) $\min \quad f(\underline{x}, \underline{p})$

subject to: $g_i(\underline{x}, \underline{p}) \leq 0, \quad i = 1, \ldots, I,$

$h_j(\underline{x}, \underline{p}) = 0, \quad j = 1, \ldots, J.$

In this model, the vector $\underline{x}$ is a vector of design variables; that is, these are the variables that represent the freedoms of choice that the design engineer has available. The vector $\underline{p}$ is a vector of model parameters.

Johnson and Benson [8], [9], have decomposed the design problem by treating a subset of variables in the design problem as parameters ($\underline{p}$) and optimizing over the remaining variables. The original subset of "parameters" are then released and optimized.

Sobieszczanski-Sobieski, et.al., [13], [14], have used *linear decomposition* in an attempt to simplify the large nonlinear models that frequently represent a design problem. This decomposition is accomplished by computing the sensitivity of the global solution to changes in subproblem solutions [15], and representing these changes by first order approximations. The ultimate advantages of this approach result from the explicit inclusion of subproblem constraints in the global optimization model and a reduction in the number of variables in the global model. In recent work [6], Haftka has attempted to improve upon the linear decomposition method by removing the difficulties

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encountered due to discontinuities in the derivatives that migrate upwards through the hierarchy.

Several authors have suggest the use of knowledge-based systems to aid in the design construction. These approaches can be dichotomized by the type of knowledge required in the system and the use of that knowledge. In one case, the knowledge is used for optimization [1], [2]. Systems of this type can be thought of as addressing the more general issue of how knowledge can be used to aid in the search of optimal solutions for mathematical models that are characterized by nonlinearities. Surprisingly, the use of knowledge specific to the design domain has not been used to full advantage in these systems.

The other type of knowledge-based system contains knowledge specific to design, but does not emphasize optimization [3], [4]. While these systems can produce quality designs, the designs are based on heuristics and are therefore best suited for producing designs that meet stated specifications.

3. GOAL DECOMPOSITION

In this paper we will examine a method of decomposition that makes use of goal programming. One of the primary motivating factors for this approach is that it is felt that the employment of goals instead of "pure constraints" conforms closely to the type of decision making currently used in the initial design process. Moreover, the decision support architecture which is examined may take advantage of a variety of other modeling techniques currently available; i.e., the Sobieszczanski-Sobieski

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approach, knowledge-based models, etc.

In what follows, we will use the term *objective* to mean a state that a decision maker has identified as desirable to attain. The term *attribute* is used to describe a measure which can be used to determine the degree to which the desirable state has been achieved. We will assume throughout this paper that with sufficient perseverance, a set of attributes may be determined that will measure the objective to the satisfaction of the decision maker.

3.1 Decision Variables

One of the fundamental characteristics that differentiates the goal decomposition approach from the methods of decomposition that have been found in the mechanical/aerospace engineering literature is the treatment of design variables. Consider once again the design hierarchy depicted in Figure 1. Imagine that an optimization has been performed at level 1. Information has been passed down from level 1 to level 2 via the vector $\underline{u}$. This vector contains the attributes of the subcomponent being designed at level 2 that are desired from the perspective of the component being designed at level 1. As a result, the vector $\underline{u}$ describes what it is that we want built (the subcomponent at level 2).

The optimization at level 2 in t.. 11 describe how we will build that subcomponent by furu..r refinement of the subcomponent into its major subcomponents. The design variables at level 2 are a vector $\underline{w}$ that contain the desired attributes of the subcomponent at the 3rd level of the hierarchy. These values

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are determined from an optimization that will choose $\underline{w}$ so that the attributes of level 1 ($\underline{u}$) are met as closely as possible while not violating any constraints that are placed on $\underline{w}$.

The goal programming formulation at level i, $i = 1, \ldots, I-1$, may be described as

$$(F2) \qquad \min \Sigma \quad b_k^+ d_k^+ + b_k^- d_k^- \qquad\qquad (1)$$

such that:

$$f_k(\underline{w}) - d_k^+ + d_k^- = u_k, \quad k = 1, \ldots, K, \qquad (2)$$

$$\underline{w} \in \Omega. \qquad\qquad (3)$$

As described above, the value of u_k in (2) was determined from a goal optimization at level $(i-1)$, $i = 2, \ldots, I-1$. In the case that the value of u_k was generated at the 0th level (the top level), some other multiobjective technique might have been used to determine the desired values of the attributes of the top level.

The values b_k^+ and b_k^- are weights assigned to the deviations d_k^+ and d_k^-, respectively, which are the amount by which we have exceeded or fallen short of the kth goal. The function $f_k(\underline{w})$ is the link between the subcomponent attributes and the component attribute u_k.

The actual value of the kth attribute that can be achieved in (P2) is the value of $f_k(\underline{w})$ in equation (2) (recall that the actual value that we would like to achieve is u_k^*). The value of the kth attribute achieved, $f_k(\underline{w}^*)$, where $\underline{w}^*$ is the optimal value of $\underline{w}$ in (P2), may not equal u_k^* for two reasons: (1) The goal set designated by equations (2) may be overly demanding and therefore

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all of the goal may not be achievable; (2) The space $\cap$ in equation (3) may constrain the problem from achieving all of the goals.

The goal decomposition algorithm can be stated as follows:

A1. Optimize the attributes of level i, i = 1,...,I-1, by mathematically relating those attributes to attributes of the subcomponents (level (i-1)).

A2. Using the values of the level i attributes obtained in the optimization at level (i-1) as goals, optimize the attributes of level (i+1), i = 1,...,I-1.

A3. If necessary, perform a reoptimization at level i by passing constraints on the attributes from level (i+1) to level i.

Several iterations between levels (i+1) and i may be necessary to obtain a satisfactory design. The constraints that are added to the level i model in step A3 are needed to cause a reallocation of resources. An example of the addition of these constraints is shown in the first application of Section 4.

Since all of the goals in (P2) may not be met, the weighting b_k^+ and b_k^- in the objective function will tend to dictate which goals are satisfied. Next we will show how these weighting may be selected so that the pursuit of the goals at level i is consistent with the desired design attributes at level (i-1).

Theorem 1: Let (P2) describe the optimization at level i and let the following model describe the descendant optimization at level (i+1):

(P3) $\min \sum c_j^+ e_j^+ + c_j^- e_j^-$ (4)

such that:

$$g_j(\underline{x}) - e_j^+ + e_j^- = w_j^*, \quad j = 1, \ldots, J, \quad (5)$$

$$\underline{x} \in X. \quad (6)$$

Then, to first order approximation, the values of c_j^+ and c_j^- should be chosen according to the following algorithm:

(B1)   For each $f_k(\underline{w}*)$ in equation (2), use the following rules to compute new constants a_k^+ and a_k^-:

(B1.1) If $f_k(\underline{w}*) = u_k$, let $a_k^+ = b_k^+$, $a_k^- = b_k^-$;

(B1.2) If $f_k(\underline{w}*) > u_k$, let $a_k^+ = b_k^+$, $a_k^- = -b_k^+$;

(B1.3) If $f_k(\underline{w}*) < u_k$, let $a_k^+ = -b_k^-$, $a_k^- = b_k^-$.

(B2)   Form the values c_j^+ and c_j^- in equation (4) by

$$c_j^+ = \sum_k a_k^+ \, \partial f_k / \partial w_j \, (\underline{w}*), \quad (7)$$

$$c_j^- = \sum_k a_k^- \, \partial f_k / \partial w_j \, (\underline{w}*). \quad (8)$$

Proof:

Consider the first order Taylor's series expansion of $f_k(\underline{w})$ around $\underline{w}*$:

$$f_k(\underline{w}) - f_k(\underline{w}*) \doteq \sum \partial f_k / \partial w_j \, (\underline{w}*) (w_j - w_j^*). \quad (9)$$

The term on the left hand side of (9) is the amount that the optimal value $f_k(\underline{w}*)$ has been missed by the selection of $\underline{w}$. The right hand side of (9) contains the term $(w_j - w_j^*)$, which is the amount by which the level $(i+1)$ optimization was unable to meet the goal w_j^* that was passed down from the level i optimization. Thus equation (9) depicts the influence that the

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failure to meet goal w_j^* at level $(i+1)$ has on the optimization at level i. If, for example, $f_k(\underline{w}^*) > u_k^*$, the optimal value of $\underline{w}^*$ caused the attribute represented by $f_k(\underline{w}^*)$ at level i to be exceeded, and $d_k^+ > 0$, $d_k^- = 0$ in equation (2). If the value of $\underline{w}$ realized from

$$w_j = g_j(\underline{x}), \quad j = 1,\ldots,J,$$

does not equal w_j^* (the goal) but instead exceeds w_j^*, then the value of the objective function at level i will change by

$$b_k^+ [f_k(\underline{w}) - f_k(\underline{w}^*)],$$

(here we assume that $\underline{w}$ is near $\underline{w}^*$ so that $f_k(\underline{w}) > u_k^*$).
But this is just the right hand side of (9) multiplied by the appropriate weight from the level i optimization. Multiplying the right hand side of (9) by b_k^+ yields the result for the special case that $f_k(\underline{w}^*) > u_k^*$ (number 2, part A of the algorithm). The other parts of the algorithm can be determined from similar arguments.

3.2 Refinements to the Goal Decomposition Algorithm

(1) If there are no constraints in equation (3) that connect w_j's from different subproblems, then a feasible design at level i may be assured easily at level $(i+1)$ by adding the constraint

$$g_j(\underline{x}) \in \Omega$$

to problem (P3).

(2) If the optimization at level $(i+1)$ does not meet the goals $\underline{w}^*$, then a reoptimization at level i may generate a better (or feasible) solution when taking into account the best values

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of the attributes that could be achieved at level (i+1). In this case, the following cuts may be added to equation (3):

(A) If $g_j(\underline{x}*) = w_j*$, then add no cuts;

(B) If $g_j(\underline{x}*) > w_j*$, then add the cut

$$w_j > g_j(\underline{x}*);$$

(C) If $g_j(\underline{x}*) < w_j*$, then add the cut

$$w_j < g_j(\underline{x}*).$$

(3) The problem that a multipurpose component (a component with multiple parent nodes in the decomposition network) causes can be coordinated in this process by adding separate goals from each parent optimization. For example, if a multipurpose component is passed down the goal $w_1* = 1$ from one parent optimization and $w_1* = 2$ from the other parent optimization, then the following two goals should be added to the subproblem:

$$g_1(\underline{x}) + d_1^- - d_1^+ = 1,$$
$$g_1(\underline{x}) + d_2^- - d_2^+ = 2,$$

where $w_1 = g_1(\underline{x})$. The coefficients in the objective function will help determine which of these contradicting goals should dominate.

4. APPLICATIONS

4.1 A Conceptual Application

The first system design that will be optimized using goal decomposition is depicted in Figure 2a. We begin with this conceptual system to illustrate the techniques that may be employed to model system cost and reliability. Similar models may be built to include attributes measuring system performance,

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supportability, and producibility.

The network decomposition of this system is shown in Figure 2b. At level 1, the system is viewed as consisting of two components in series, as indicated by the dotted line in Figure 2a. We will assume that Component 1 may be viewed as consisting of two identical components in parallel, while component 2 consists of two components in series.

We shall also assume that an optimization has been performed at level 0 that specified the target values of the reliability and cost of the entire system to be $R^* = .995$ and $C^* = \$2000$. Let:

R_i = Reliability of component i, i = 1,2,

       = Probability that the component operates without failure over a specified time interval,

C_i = Cost of component i, i = 1,2,

R_{ij} = Reliability of subcomponent (i,j), i = 1,2,   j = 1,2,

C_{ij} = Cost of subcomponent (i,j), i = 1,2, j = 1,2.

It is also assumed that R_1 and R_2 must be at least .95 and .92 respectively, and that the total budget for the system is \$2600.

In order to build a mathematical model of the system, it is first necessary to determine how reliability influences cost. Suppose that by experience it is known that the reliability of the subcomponents is related to the cost of the subcomponents in the following way:

$$C_{11} = -200 \ln\{1 - R_{11}(2 - R_{11})\}, \tag{10}$$

$$C_{12} = C_{11}, \tag{11}$$

$$C_{21} = 1.2/(1 - R_{21}), \tag{12}$$

$$C_{22} = 1/(1 - R_{22}).\tag{13}$$

Notice that in each of these cases the cost of the subcomponent increases rapidly as the reliability approaches 1.

The development of the cost functions C_1 and C_2 for an optimization at level 1 in Figure 2b presents two significantly different problems. Component 1 consists of two identical components in parallel; and, therefore, the cost function may be written simply as

$$\begin{aligned} C_1 &= C_{11} + C_{12} \\ &= -400 \ln\{1 - R_1\}, \end{aligned}\tag{14}$$

since

$$R_1 = R_{11}(2 - R_{11}).\tag{15}$$

On the other hand,

$$C_2 = C_{21} + C_{22}\tag{16}$$

cannot be written as a simple function of

$$R_2 = R_{21}R_{22}.\tag{17}$$

Consequently, the developer of a model has two alternative approaches: (1) Include in the level 1 optimization model mathematical expressions that involve R_{21} and R_{22}, which are level 2 attributes; or (2) Estimate the cost function C_2 as a function of R_2. The former alternative regresses toward the problem that prompted decomposition in the first place: the model begins to grow larger because we have decided to include too much detail in the model at the higher levels in the network. The second alternative risks the loss of some information in the estimation process but maintains the spirit of decomposition.

For example, using equations (12) and (13), the following

table of costs for alternative designs may be easily constructed:

R_{2i} (i = 1,2)	C_{21}	C_{22}
.9	\$ 12	\$ 10
.925	16	13.3
.95	24	20
.975	48	40
.99	120	100
.995	240	200
.999	1200	1000

Table 1--C_{21} and C_{22}

Using equation (17) and different combinations of R_{21} and R_{22}, values of C_2 may be computed from Table 1. Next, a least square estimation technique may be used to develop an estimate of C_2 as a function of R_2. Such a model was developed for this example from 15 combinations of R_{21} and R_{22} which yielded the cost function:

$$C_2 = 154.2 + 4.8(1 - R_2)^{-1}. \tag{18}$$

The optimization model at level 1 may now be written as:

(M1) $\qquad$ min $z = 5000d_1^- + d_2^+$

such that

$$R_1 R_2 - d_1^+ + d_1^- = .995$$

$$C_1 + C_2 - d_2^+ + d_2^- = 2000$$

$$C_1 + C_2 \leq 2600$$

$$d_1^+ d_1^- = 0$$

$$d_2^+ d_2^- = 0$$

$$.95 \leq R_1 \leq 1$$

$$.92 \leq R_2 \leq 1$$

as well as equations (14) and (18) are satisfied and all of the variables in the model are non-negative.

The coefficients in the objective function of (M1) reflect the trade-offs that were judged reasonable by the decision makers at level 0 and have been assumed for this example. The optimal solution to (M1) generated by GINO [10] is:

$$z = 158.187 \qquad d_2^- = 0 \qquad C_2 = 455.48$$

$$d_1^- = .032 \qquad d_2^+ = 0 \qquad R_1 = .97896$$

$$d_1^+ = 0 \qquad C_1 = 1544.52 \qquad R_2 = .98407$$

4.1.1 Level 2 Optimization Models

To generate a model that optimizes the design of the subcomponents, we need to use the values of C_i and R_i, $i = 1, 2$, that were computed in the optimal solution to (M1). In addition, the results of Theorem 1 are needed to search for solutions that are consistent with the level 1 optimization. The models (M11) and (M12) for components 1 and 2 respectively and their solutions may be found in appendix A.

(M11) and (M12) contain constraints that are local to the subcomponent design as depicted by equation (6). For example, in (M11), $R_{11} \leq .8$. The solutions for C_i and R_i in (M1) are used as the goals in (M11) and (M12). Also notice that the constraints $0.95 \leq R_1 \leq 1$ have been included in (M11) as suggested in (1) of section 3.2.

Table 2 compares the values of R_i and C_i, $i = 1, 2$, that were generated in the solutions of (M1) with (M11) and (M12). It is clear from this table that the largest value of R_1 that can

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possibly be generated is .96, and that \$1287.55 is the largest budget needed; otherwise, additional dollars would have caused the solution of R_1 in (M11) to be larger.

Variable	Derived from Level 1	Achieved at Level 2
R_1	.97896	.96
C_1	1544.52	1287.55
R_2	.98407	.97752
C_2	455.48	455.48

Table 2--Attribute Goals and Values Realized
in (M11) and (M12)

If the constraints $R_1 \leq .96$ and $C_1 \leq 1287.55$ are added to (M1), the new solution to (M1) becomes

$$z = 216.272 \qquad d_2^- = 0 \qquad C_2 = 712.45$$

$$d_1^- = .043 \qquad d_2^+ = 0 \qquad R_1 = .96$$

$$d_1^+ = 0 \qquad C_1 = 1287.55 \qquad R_2 = .9914$$

This solution is, in fact, the globally optimal design; i.e., the design that would be generated if we had built one model that had contained all of the constraints on the component and subcomponent designs.

In summary, in order to generate the globally optimal design, it was necessary to pass information downward through the decomposition network that described the desired attributes of the system. Information in the form of constraints was passed upward through the network that indicated the best achievable

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values of the attributes. A reoptimization was performed that reallocated the resources (in this case budget dollars) to improve the design. Further optimization iterations, in general, would continue to improve the design.

4.2 A Second Application

Consider the problem of designing a water pumping system and a structural support for the pumping system. As shown in Figure 3a, it is assumed that the water pumping system consists of two parallel pumps that draw water from a lower reservoir and pump the water into a reservoir that is 150 feet above the lower reservoir [16]. The pumping system is to be positioned on a 30 foot reinforced concrete beam of depth 30 inches as discussed in [9]. The decomposition network for the pump/beam system is shown in Figure 3b.

Assume that a budget of $950 may be spent on the project but that the desired budget is $900 (as determined in an optimization at level 0). A goal of 200 feet for the head loss has been established to take into account the loss of energy due to friction in the pipes. This head loss must be, at a minimum, 160 feet. In addition, a goal of 6480 in k of flexural strength is desired in the beam. Define:

C_1 = Cost of the pump subsystem,

C_2 = Cost of the beam subsystem,

H = Head loss,

S = Flexural strength of the beam.

Prior experience has shown that C_1 and C_2 increase with H

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and S in the following way:

$$C_1 = .01125H^2 + .875H, \qquad (19)$$

$$C_2 = .05787S. \qquad (20)$$

Assuming once again that the optimization at level 0 has supplied the coefficients in the objective function that reflect the trade-offs that would be considered appropriate at the top level, the pump/beam optimization model can be written as:

(M2)   min $z = d_1^+ + 5d_2^- + .0926d_3^-$

such that

$$C_1 + C_2 - d_1^+ + d_1^- = 900$$

$$H \qquad - d_2^+ + d_2^- = 200$$

$$S \qquad - d_3^+ + d_3^- = 6480$$

$$C_1 + C_2 \qquad \leq 950$$

$$H \qquad \geq 160$$

$$d_i^+ d_i^- \qquad = 0, \quad i = 1, 2, 3,$$

equations (19) and (20) hold and all variables in the model are nonnegative. The optimal solution to (M2) is:

$z = 96.875$	$d_2^+ = 0$	$C_2 = 375$
$d_1^- = 0$	$d_3^- = 0$	$H = 183.34$
$d_1^+ = 13.59$	$d_3^+ = 0$	$S = 6480$
$d_2^- = 16.66$	$C_1 = 538.59$	

4.2.1 Level 2 Models

Next consider the optimization of the pump subject to local constraints. In this example, assume that the headloss may be expressed in terms of the flow rates (cubic feet/second), F_1, through pump i, i = 1,2:

$$H = A + 30F_1 - 6F_1^2 \qquad (21)$$

$$H = B + 20F_2 - 12F_2^2 \qquad (22)$$

In these expressions, A and B are parameters which specify attributes of the pumps. The selection of the pumps is restricted to

$$50 \le A \le 130 \qquad (23)$$

$$200 \le B \le 220. \qquad (24)$$

The amount of flow in each pump is restricted to be at least 2 cubic feet/second.

Let C_{1j} be the cost of pump j, $j = 1,2$. The cost of the pumps is related to the flow rate via

$$C_{11} = 100 \ F_1 \qquad (25)$$

$$C_{12} = 120 \ F_2. \qquad (26)$$

The total budget for the pump subsystem will be at least \$525. The optimization model (M21) for the pump subsystem and the optimal solution is given in appendix B.

The beam subsystem has design variables W and V, which represent the width of the beam and the cross sectional area of the reinforcing steel. For this example, it is assumed that the reinforcing steel has standard sizes that come in .25 inch increments. The flexural strength, S, of the beam is related to W and V by

$$V - .2458 \ V^2/W \ge S/1080. \qquad (27)$$

The unit cost of the concrete is \$.02/square inch/lineal foot and the unit cost of steel is \$1.00/square inch/lineal foot. The length of the reinforcing steel is slightly less than the length of the beam (29.4 feet), so that the cost of the steel is given by 29.4V. The cost of the concrete is given by

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.02 x (cross-sectional area of the beam) x 30

$= .02$ x (W x 30) x 30

$= 18$ W.

The width of the beam must be at least 8 inches to provide sufficient support to the pump subsystem. Once again using Theorem 1, the optimization model (M22) for the beam subsystem may be formulated and solved as shown in appendix C.

The value of V in the solution of (M22) is not a integral multiple of .25; as a result, this solution is not truly feasible given the problem constraints. In order to achieve the optimal feasible solution, two new models (M23) and (M24) may be formed from (M22) by adding the constraints $V \leq 7.75$ in the first case and $V \geq 8$ in the second case. Model (M24) has no feasible solution, while the optimal solution to (M23) is given by:

$$z = 12.69 \qquad \bar{e}_2 = 3.15 \qquad S = 6376.32$$
$$e_1^- = 103.05 \qquad e_2^+ = 0 \qquad W = 8$$
$$e_1^+ = 0 \qquad C_2 = 371.85 \qquad V = 7.75$$

This solution is therefore the optimal attributes for the design of the beam subsystem. The solution to (M21) and (M23) may be used to generate constraints in order to further constrain the solution to (M2). This would begin a new iteration of the goal decomposition algorithm. However, a designer may decide that the design achieved by combining the solution to (M21) and (M23) to be satisfactory.

5. CONCLUDING REMARKS

This paper has attempted to develop a broader perspective

of the engineering design problem by distinguishing the type of optimization problems that occur at the top and lower levels of the design hierarchy. One key feature of middle and lower level optimizations is that subsystem optimization does not correspond directly to performance, cost, etc. optimization. A goal decomposition method was described and illustrated for subcomponent optimization. This decomposition method had a natural objective function that tended to cause the solution of the subproblem to drive the solution of the optimization one level higher to its best possible feasible solution.

The architecture of the decision support system described using goal decomposition readily enhanced the iterative nature of the design process. In the first example shown in this paper, a single reoptimization found the best global solution to the design problem once additional information was supplied to the level 1 model by subsystem optimizations.

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(M11) min $z = -4920.34e_1^+ + 4920.34e_1^- + e_2^+$

such that

$$R_1 - e_1^+ + e_1^- = .97896$$

$$C_1 - e_2^+ + e_2^- = 1544.52$$

$$R_1 = R_{11}(2 - R_{11})$$

$$R_1 \geq .95$$

$$R_1 \leq 1$$

$$R_{11} \leq .8$$

$$e_i^+ e_i^- = 0 \quad i = 1,2$$

as well as equations (10), (11), and (14) and all variables are
nonnegative.

Optimal solution:

$z = 93.78$ $e_2^+ = 0$ $C_1 = 1287.55$

$e_1^+ = 0$ $e_2^- = 256.97$ $R_{11} = .8$

$e_1^- = 256.97$ $R_1 = .96$ $C_{11} = 643.78$

(M12) min $z = -4894.8e_1^+ + 4894.8e_1^- + e_2^+$

such that

$$R_2 - e_1^+ + e_1^- = .98407$$

$$C_2 - e_2^+ + e_2^- = 455.48$$

$$R_2 = R_{21}R_{22}$$

$$R_2 \geq .95$$

$$R_2 \leq 1$$

$$R_{21} \geq .9$$

$$R_{21} \leq .98$$

$$R_{22} \geq .99$$

$$R_{22} \geq 1$$

$$e_i^+ e_i^- = 0 \quad i = 1,2$$

as well as equations (12), (13), and (16) and all variables are nonnegative.

Optimal solution:

$z = 32.04$ $\quad$ $e_2^+ = 0$ $\quad$ $R_{22} = .99747$

$e_1^+ = 0$ $\quad$ $e_2^- = 0$ $\quad$ $C_2 = 455.48$

$e_1^- = .00655$ $\quad$ $R_2 = .97752$ $\quad$ $C_{21} = 60$

$\quad\quad\quad\quad\quad\quad\quad R_{21} = .98$ $\quad$ $C_{22} = 395.48$

(M21)   min $z = -5e_1^+ + 5e_1^- + e_2^+ - e_2^-$

such that

$$H \quad - e_1^+ + e_1^- \qquad = \quad .98407$$

$$C_1 - e_2^+ + e_2^- \qquad = \quad 538.59$$

$$C_1 \qquad = \quad C_{11} + C_{12}$$

$$C_1 \qquad \geq \quad 525$$

$$F_1 \qquad \geq \quad 2$$

$$F_2 \qquad \geq \quad 2$$

$$e_i^+ e_i^- \qquad = \quad 0 \quad i = 1,2$$

as well as equations (21)-(26) and all variables are nonnegative.

Optimal solution:

$z = 73.25$	$C_1 = 525$	$F = 4.71$
$e_1^+ = 0$	$C_{11} = 199.54$	$F_1 = 2$
$e_1^- = 17.37$	$C_{12} = 325.46$	$F_2 = 2.71$
$e_2^+ = 0$	$A = 130$	$H = 165.97$
$e_2^- = 13.59$	$B = 200$	

(M22)   min $z = .0926e_1^- - e_2^+ + e_2^-$

such that

$$S \quad - e_1^+ + e_1^- \qquad = 6480$$

$$C_2 - e_2^+ + e_2^- \qquad = 375$$

$$C_2 \qquad\qquad\qquad = 29.4V + 18W$$

$$W \qquad\qquad\qquad \geq 8$$

$$e_i^+ e_i^- \qquad\qquad = 0 \quad i = 1,2$$

as well as equation (27) and all variables are nonnegative.

Optimal solution:

$z = 12.69$	$C_2 = 375$
$e_1^+ = 0$	$S = 6437.17$
$e_1^- = 103.05$	$W = 8$
$e_2^+ = 0$	$V = 7.857$
$e_2^- = 3.15$	

DECISION
VARIABLES

U

W

X
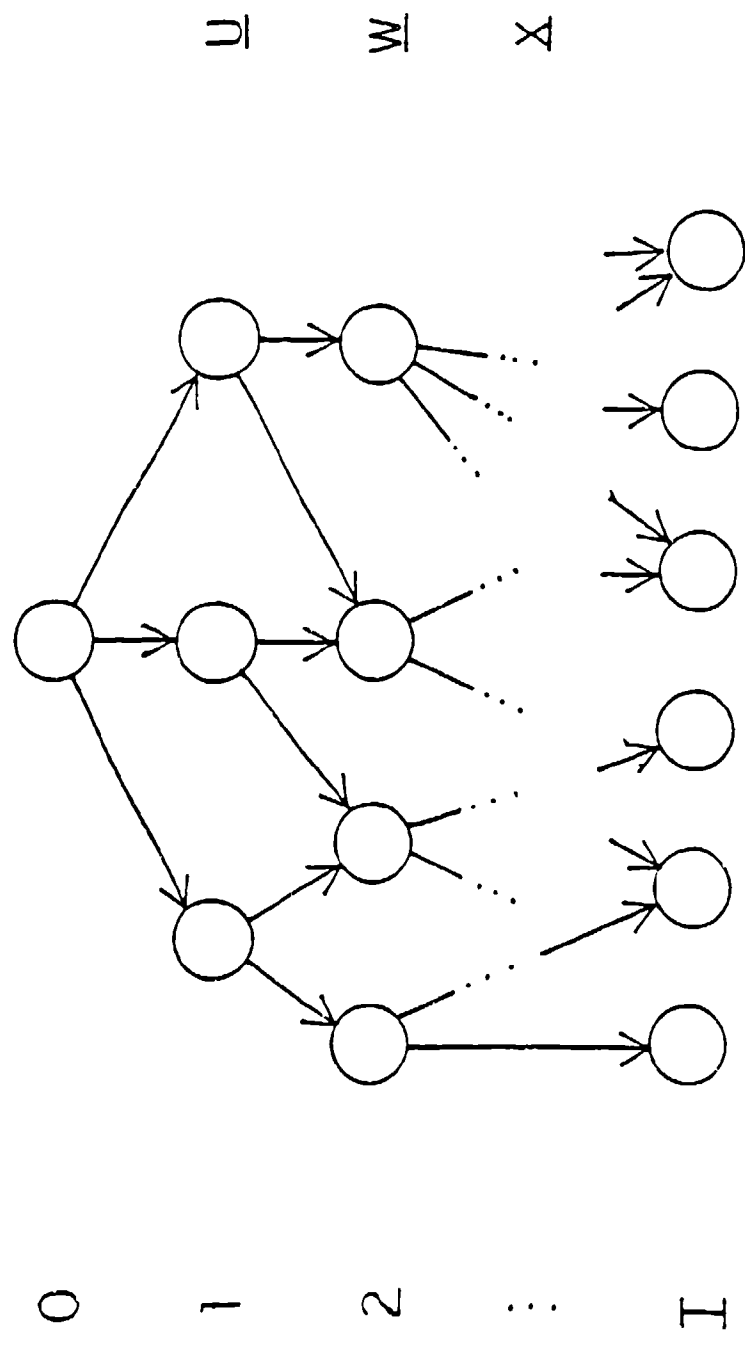
Figure 1--Decomposition Network

LEVEL

0

1

2

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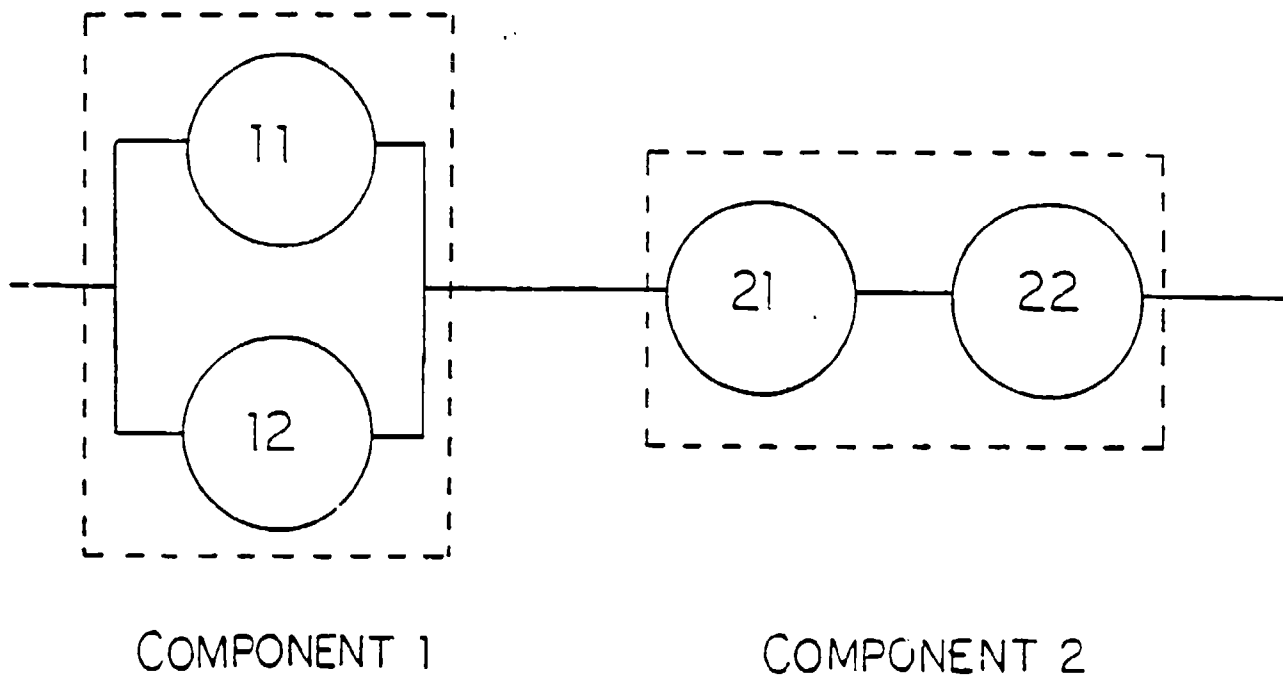
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COMPONENT 1   COMPONENT 2

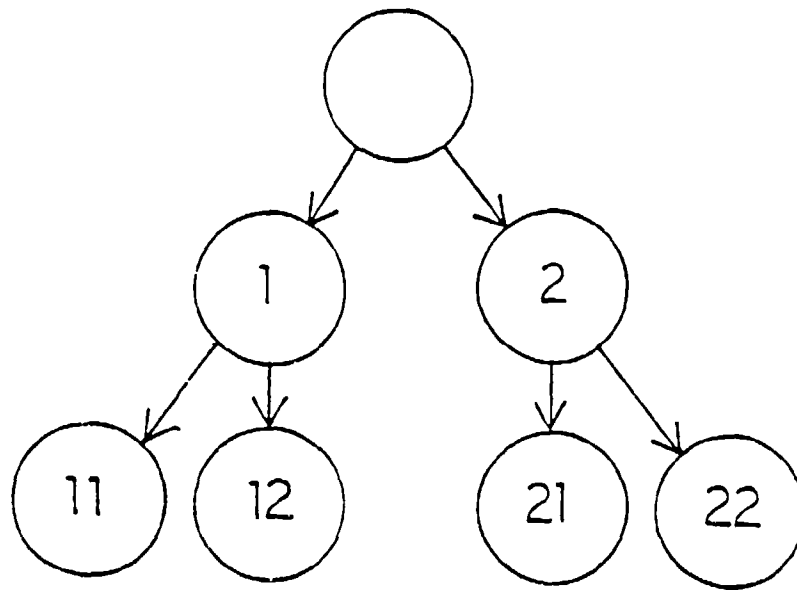Figure 2a—A Conceptual System

LEVEL

0

1

2

Figure 2b—Network Decomposition
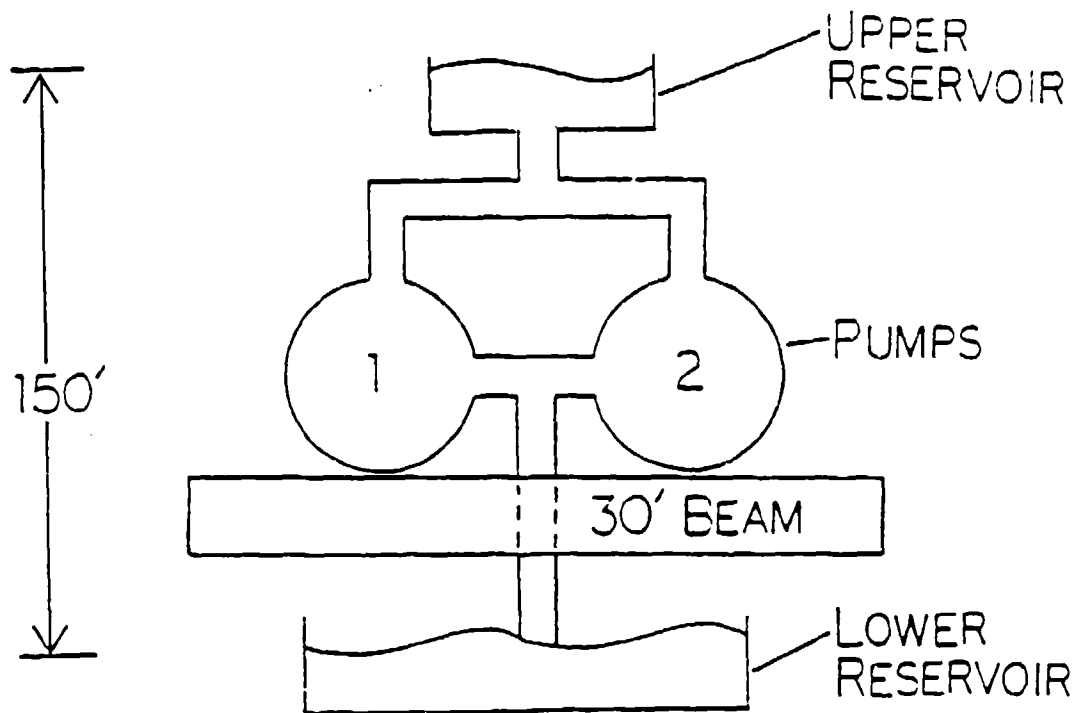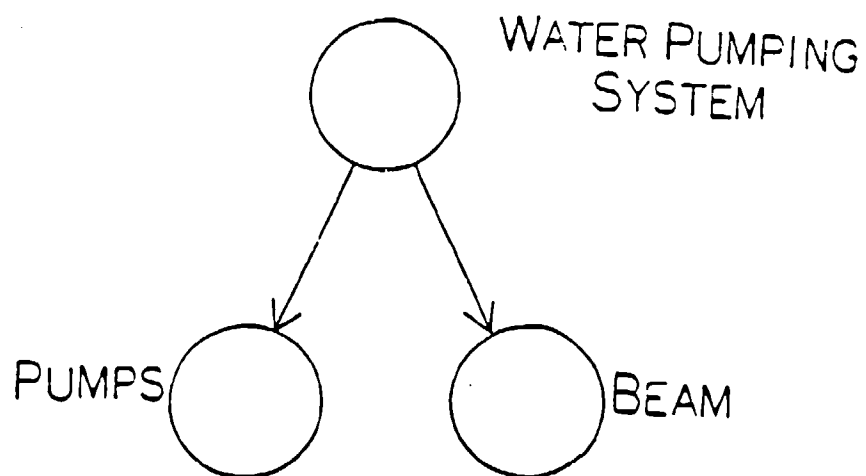of the Conceptual System

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Figure 3a--Water Pumping System

Figure 3b—Network Decomposition of the
Water Pumping System